Deutsch-Jozsa properties:

Deutsch-Jozsa oracle functions:

* **Balanced function:**

A function is *balanced iff:*

For *exactly half* of all *: = 0*

For *exactly half* of all *: = 1*

* **Constant function:**

A function is *constant iff:*

*: = 0 : = 1*

* **Inputs**: Two oracles and of size and that implements and respectively.
* **Precondition**: and are ***balanced***:
* **Operation**:

* **Postcondition**:

2)

* **Inputs**: Two oracles and of size and that implement and respectively.
* **Precondition**: and are ***constant***:
* **Operation**:

* **Postcondition**: (constant)

3)

* **Inputs**: Two oracles and of size and that implement and respectively.
* **Precondition**: is ***constant*** and is ***balanced***:
* **Operation**:

* **Postcondition**:

4)

* **Inputs**: Two oracles and of size and that implement and respectively.
* **Precondition**: and are ***balanced*** and :
* **Operation**:

* **Postcondition**: (constant)

5)

* **Inputs**: Two oracles and of size and that implement and respectively.
* **Precondition**: and are ***balanced*** and :
* **Operation**:

* **Postcondition**:

**Only balanced if merged oracle control gates line up, otherwise const**

6)

* **Inputs**: Two oracles and of size and that implement and respectively.
* **Precondition**: and are ***constant***:

**Operation**:

* **Postcondition**: (constant)

7)

* **Inputs**: Two oracles and of size and that implement and respectively.
* **Precondition**: is ***constant*** and is ***balanced***:

**Operation**:

* **Postcondition**:

8)

* **Inputs**: One oracle size .

A vector

**Where**

* **Precondition**: is ***constant***:

**Operation**:

* **Postcondition**:

9)

* **Inputs**: One oracle size .

A vector

**Where**

* **Precondition**: is ***balanced***:

**Operation**:

* **Postcondition**:

(if we remove the right wire, then we can get , still thinking about how to define it)

**Vertically Merging oracles*:***

*A diagram of a mathematical equation

Description automatically generated with medium confidence*

Let two Deutsch-Jozsa oracles and of size and respectively, where the lower register qubits are at wire indexes and .

To vertically merge the oracles, we create a new circuit of register size and permute the wires:

* Oracle upper register wires to indexes in
* Oracle upper register wires to indexes
* Oracle and lower register wires at and to

**When applying to two constant oracles** , **the resulting oracle is also** **constant:**

Constant oracles return the same output regardless of input, this can be easily accomplished by inserting an oracle that implements identity or applies a Pauli-X gate in the lower register .

**Example:**

Due to the wire permutation, the gates applied to the lower register wire on both , will be applied to the lower register wire on , meaning that the Pauli-X from will be in the lower register in , resulting in a constant output.

Consider the truth tables for , for and :

|  |  |  |
| --- | --- | --- |
| Upper register input | lower register output | lower register output |
|  |  |  |
|  |  |  |
|  |  |  |
|  |  |  |

|  |  |
| --- | --- |
| Upper register input | lower register output |
|  |  |
|  |  |
|  |  |
|  |  |

**Where:** is true if oracle is **balanced**

**Where:** is true if oracle is **constant**

**Horizontally merging oracles**:

A diagram of a mathematical equation

Description automatically generated

Let two Deutsch-Jozsa oracles and of size and respectively, where the lower register qubits

To horizontally merge the oracles, we create a new circuit of register size and permute the wires:

* Oracle upper register wires to indexes in
* Oracle upper register wires to indexes in
* Oracle and lower register wires at and to

(the control is in the same locations)

(Unless the control is in the same locations)

**Where:** is true if oracle is **balanced**

**Where:** is true if oracle is **constant**

**(just 1 wire)**

**Removing wires**:

Let one Deutsch-Jozsa oracle of size where the lower register qubit is at wire index .

To remove wires from an oracle, we create a new circuit of register size and permute the wires:

* Oracle upper register wires to indexes
* Oracle lower register wire at to index

if right wire is not removed

if the right wire is removed

**Where:** is true if oracle is **balanced**

**Where:** is true if oracle is **constant**

**Adding wires**:

Let one Deutsch-Jozsa oracle of size where the lower register qubit is at wire index .

To add wires to an oracle, we create a new circuit of register size + and permute the wires:

* Oracle upper register wires to indexes
* Oracle lower register wire at to index